

coming per minute. What is the probability that up to a minute will elapse unit 2 calls have come in to the switch board? (8)

12. (a) Given the joint density function

$$f(x, y) = \begin{cases} x \frac{(1+3y^2)}{4}, & 0 < x < 2, 0 < y < 1 \\ 0 & , \text{ elsewhere} \end{cases}$$

Find the marginal densities  $g(x), h(y)$  and the conditional density  $f(x/y)$  and evaluate  $P\left[\frac{1}{4} < x < \frac{1}{2} / Y = 1/3\right]$ .

Or

- (b) (i) Determine whether the random variables  $X$  and  $Y$  are independent, given their joint probability density function as (8)

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & , \text{ otherwise} \end{cases}$$

- (ii) If  $X$  and  $Y$  are independent random variables having density functions

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \text{ and} \\ 0 & , x < 0 \end{cases}$$

$$f(y) = \begin{cases} 3e^{-3y}, & y \geq 0 \\ 0 & , y < 0 \end{cases}$$

respectively, find the density functions of  $z = X - Y$ . (8)

13. (a) (i) Show that random process  $\{X(t)\} = A \cos t + B \sin t, -\infty < t < \infty$  is a wide sense stationary process where  $A$  and  $B$  are independent random variables each of which has a value  $-2$  with probability  $\frac{1}{3}$  and a value  $1$  with probability  $2/3$ . (8)

- (ii) Derive probability distribution of Poisson process and hence find its auto correlation function. (8)

Or

- (b) (i) Find the limiting-state probabilities associated with the following transition probability matrix.

$$\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}. \quad (10)$$